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THE DS-1 AUTONOMOUS NAVIGATION SYSTEM: AUTONOMOUS CONTROL OF LOW THRUST PROPULSION SYSTEM

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Abstract

The Deep Space-1 (DS-1) mission to be launched in 1998 will use an autonomous navigation system to guide the spacecraft on a low thrust trajectory to flybys of an asteroid and a comet. The ion propulsion system to be validated on 1) S- 1 will provide low thrust solar electric propulsion to the spacecraft and presents additional challenges to the development of the autonomous navigation system. In order to maintain a trajectory to the designated mission target bodies, the autonomous navigation system must autonomously determine the orbit of the spacecraft, and adjust the thrust profile to be implemented by the ion propulsion system to correct any deviations from the nominal spacecraft trajectory. A detailed description of the component of the autonomous navigation system that controls the low thrust profile of the ion propulsion system is presented, and examples of some tests of this system are used to illustrate its capabilities.

Introduction

The first of NASA's New Millennium technology validation missions, the Deep Space-1 (1) S-1) mission', will be used to demonstrate and validate the first completely autonomous navigation system ever used by an interplanetary mission. Among the various technologies to be validated on the 1) S-1 mission, the most important is the" use of an ion propulsion system (IPS) as the primary propulsion system of the spacecraft. The IPS provides solar electric propulsion (SEP) by accelerating ionized xenon gas through a large potential. Historically, spacecraft trajectory corrections have been performed using chemical rocket engines which provide a relatively high thrust over short (minutes to hours) durations of time. The amount of total impulse available to the spacecraft is limited by the mass of propellant that the spacecraft can carry. In contrast, SEP has the capacity to provide continuous low thrust to the spacecraft, of the order of tens of millinewtons, for durations that are as long as many months. SEP is especially beneficial to high energy interplanetary missions where large changes in the energy of the orbit of the spacecraft can be achieved with considerably less mass than a chemical propulsion system.

The low thrust provided by the IPS is the largest nongravitational force acting on the spacecraft, and errors in the pointing angle, duration, and magnitude of the thrust applied by the IPS on DS-1 are likely to be the largest cause for deviations from the nominal spacecraft trajectory. The implementation of the nominal design of the SEP thrust profile on 1) S- 1 is expected to have accuracies of the order of 1-2%. Continuous monitoring of the IPS and regular updates of the thrust pointing angles and thrust durations will be necessary to correct for deviations from the designed SEP thrust profile and spacecraft trajectory. Although redesigns of the SEP thrust profile could be computed on the ground, it would be much more efficient and advantageous to compute corrections to the designed SEP thrust profile on the spacecraft itself since these updates are expected to occur frequently. Autonomous control of the IPS on 1) S- 1 is an integral part of the autonomous navigation system.

The 1) S- 1 autonomous navigation system will use autonomous optical navigation (OPNAV) to determine the best estimated orbit of the spacecraft. This best estimate of the spacecraft state will then be used to compute the corrections to the designed SEP thrust profile that are necessary to maintain a spacecraft trajectory to the designated targets. The OP-NAV system uses a camera onboard the spacecraft to take images of the relative positions of asteroids with respect to the spacecraft. This information is then used to determine the spacecraft position and velocity using precise orbit determination techniques. More details of the DS-1 autonomous navigation system and the OPNAV system are described elsewhere[2,3,4]. This paper is devoted to describing the current strategies and algorithms that will be used by the autonomous guidance and control component of the DS-1 autonomous navigation system to adjust the designed SEP thrust profile to be implemented by the IPS in order to achieve the specific target conditions. The results from some tests used to validate this low thrust trajectory guidance and cent rol system are also discussed.

Definition of the Designed Thrust Profile

The nominal SEP thrust profile for the low thrust trajectory of DS-1 is designed prior to launch as a completely independent process to the autonomous navigation system[5]. At present, the DS-1 trajectory is being designed for an encounter with the asteroid McAuliffe, a flyby of Mars, and an encounter with the asteroid West-Kahoutek-Ikemoura (WKI). The DS-1 autonomous control system will be responsible for computing updates and small changes to the designed SEP profile. However, if the corrected SEP thrust profile becomes energetically disadvantageous for subsequent encounters, or if there are significant deviations from the designed SEP thrust profile, the ground navigation team will have opportunities to redesign the SEP profile for uplink to the spacecraft. It is likely that early redesigns will occur immediately after launch to account for orbit injection errors, and after the IF'S has been calibrated.

In order to simplify the design and control of the 1) S-1 trajectory, the designed SEP thrust profile will be split into successive planning cycles. The majority of the planning cycles will have a duration of 7 days, while plans on approach to the target encounter time will become successively shorter. This allows the autonomous navigation system to prepare, or plan, the SEP profile for upcoming plans by computing the precise orbit of the spacecraft before computing the adjusted SEP profile for the future plans that occur before encounter time. Figure 1 provides a heliocentric view in the equatorial plane of a sample DS- 1 low thrust trajectory to encounters with McAuliffe and WKI. The launch date for this trajectory is July 1, 1998, and the encounters with

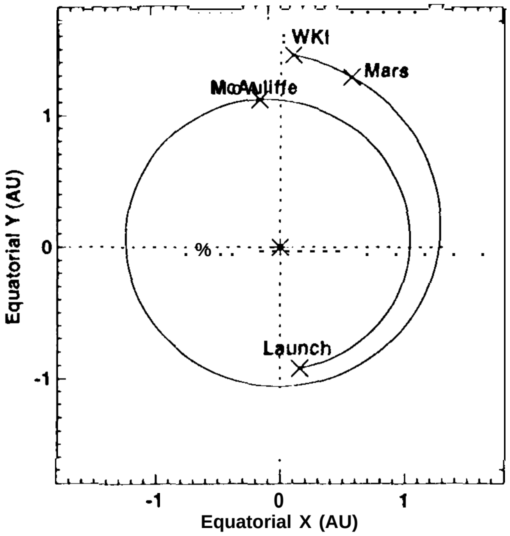

Figure 1: Sample DS-1 Trajectory to McAuliffe and West- Kahoutek-Ikemoura

McAuliffe and WKI are on January 17, 1999 and June 6, 2000, respectively.

The SEP profile for each planning cycle k, for $k = O$ to K, will be defined by a constant thrust magnitude T_k and consequently a constant mass flow rate, and a duration τ_k that the SEP thrust is applied during each plan. The 11'S thrust pointing vector in each plan is specified by the time dependent pointing angles of right ascension $\alpha(t)$, and declination $\delta(t)$, which are each defined by first order polynomials of time in each plan.

$$\alpha(t) = \alpha_k + \dot{\alpha}_k(t - t_k) \; ; \; t_k \leq t < t_k + \tau_k \quad (1)$$
$$\delta(t) = \delta_k + \dot{\delta}_k(t - t_k) \; ; \; t_k \leq t < t_k + \tau_k \quad (2)$$

In addition, a particular *duty cycle* D is imposed on the SEP profile of the low thrust trajectory when it is designed, where the duty cycle specifies the maximum duration that the IPS is permitted to thrust in each planning cycle. A constant duty cycle is usually defined for the entire SEP thrust profile. Here, reference will also be made to SEP segments, where an individual SEP segment refers to the combination of SEP plans where the IPS is thrusting continuously except for the time at the end of a SEP plan where the IPS is not thrusting only because of the imposed duty cycle limitations. This means that all of the plans except for the last plan in any particular SEP segment will have a thrust duration that is exactly at the specified duty cycle limit. Only the last plan κ of each SEP segment is permitted to have a thrust duration that is free to range from zero duration to the duration available from the specified duty cycle limit. Given the start time t_k of each planning cycle

k in a SEP segment, the implicit constraint on the durations that the 11'S is permitted to thrust in each plan of a particular SEP segment is as follows.

$$\tau_k = D(t_{k+1} - t_k) \text{ when } k \# \kappa \tag{3}$$

$$0 \leq \tau_\kappa < D(t_{\kappa+1} - t_\kappa) \tag{4}$$

All SEP plans that arc not part of a SEP thrusting segment will have a thrust duration of τ_k = O.

The nominal 11 S- 1 SEP profile is designed to allow approximately 8% of the duration in each planning cycle to be devoted to telecommunications with ground operations, and to taking the images of the asteroids that are used as beacons by the OPNAV system for the autonomous orbit determination of the spacecraft. Due to attitude constraints on the spacecraft the 11'S cannot be operating during either of these procedures. The remaining 92% of the duration in each planning cycle is available for thrusting by the IPS. For the actual DS-1 flight the SEP profile will be designed such that the II'S will have a 92'% duty cycle. However, for the purposes of testing the autonomous navigation system, and especially the autonomous control system, trajectories with a suboptimal 85% duty cycle are currently being used. This approach is taken to ensure that trajectories with suboptimal performance from the 11'S are available for the McAuliffe and WKI encounters, but also to ensure that the autonomous control system is capable of controlling the DS-1 trajectory if the 11'S does not perform to the specified 92c% duty cycle specifications.

The 1) S- 1 trajectory shown in Figure 1 is designed to an 85% duty cycle, and the associated SEP profile between launch and the McAuliffe encounter, is shown in Figure 2. The pointing angles in each SEP segment could be considered to be continuous except for the time during the SEP plans when the IPS is not thrusting because of the specified duty cycle limit. The SEP profile for the McAuliffe encounter, shown in Figure 2, has two SEP segments. The first SEP segment begins 15 clays after launch, is approximately 10 days long, and contains 2 SEP plans. The second SEP segment begins 31 days after launch, is 100 days long, and contains 16 SEP plans. The first segment at tbe beginning of the mission is specifically designed to be used to test and calibrate tile 11's.

It should be noted that the right ascension and declination of the SEP thrust pointing vector from the last plan in each segment, α_κ and δ_κ, are extrapolated using the rates $\dot{\alpha}_\kappa$ and $\dot{\delta}_\kappa$, to the few subsequent plans which have zero IPS thrust durations. The reason for this is to provide nominal design values of the II'S thrust pointing angles in these zero

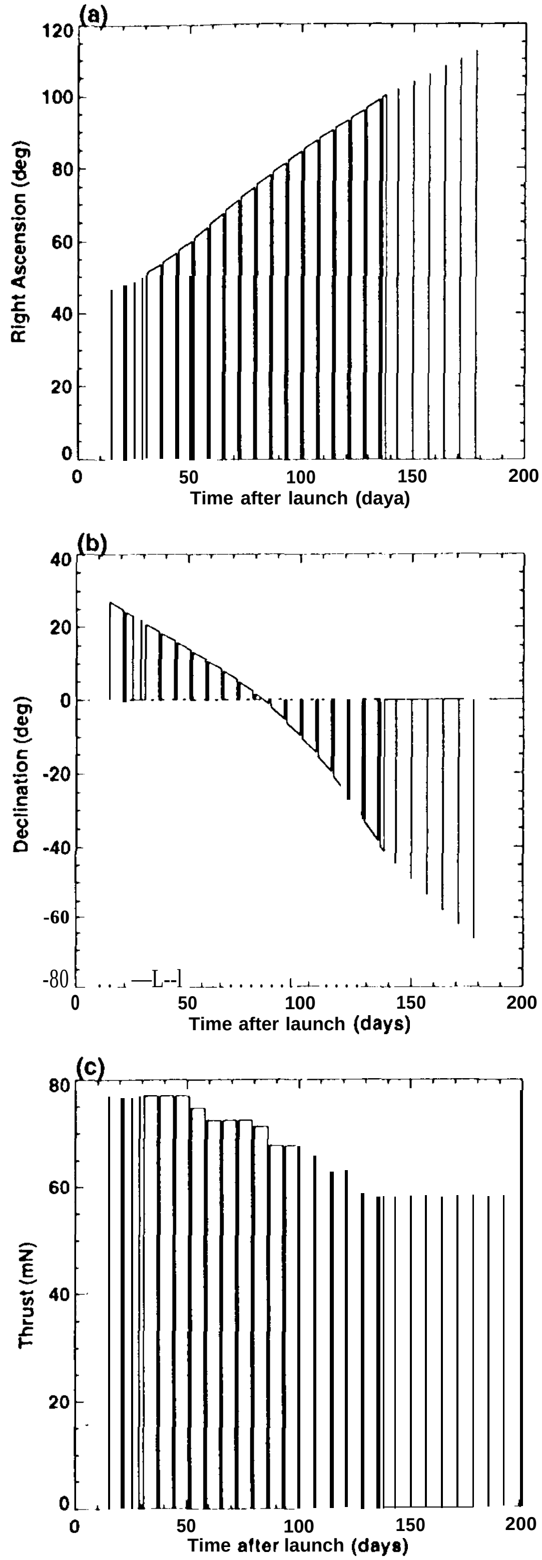

Figure 2: Right Ascension (a), [declination (b), and Magnitude (c) of SEP Thrust Pointing Vector for 1) S-1 Trajectory to McAuliffe

duration plans to allow the autonomous control system to include these plans as part of the individual SEP segments if it becomes necessary for the IPS to thrust during these plans. For example, there are six SEP plans at the end of the second SEP segment which are nominally designed with zero duration, but which could become part of the second SEP segment and be used to thrust by the IPS if necessary.

Those SEP plans that are not needed to correct the designed SEP profile then become available for trajectory control maneuvers (TCMs) for the ballistic phase of the trajectory before encounter. TCMs will be performed either by the 11'S or the hydrazine engines on DS-1, and should usually have durations of less than 12 hours if the IPS is used to perform these maneuvers.

The 1) S-1 spacecraft is severely constrained in orientation, because certain faces of the spacecraft cannot be illuminated by the Sun, and because use of the IPS requires that the solar panels face directly into the Sun. These constraints in orientation translate into constraints on the pointing angle of the 11'S thrust vector. When the SEP profile of the 1) S-1 missionisdesigned, these angular constraints on the 11'S thrust vector are specified in each plan by angles θ_k for each plan k.

Define the pointing vectors $\hat{p}'$ and $\hat{p}$ to be the thrust, pointing vectors at the beginning of each plan of the designed SEP profile, and the corrected SEP profile, respectively.

$$\hat{p}' = [\cos\delta'_k \cos\alpha'_k \quad \cos\delta'_k \sin\alpha'_k \quad \sin\delta'_k] \quad (5)$$
$$\hat{p} = [\cos\delta_k \cos\alpha_k \quad \cos\delta_k \sin\alpha_k \quad \sin\delta_k] \quad (6)$$

The primes (') are used here to indicate that the pointing vectors and angles are from the designed SEP profile. The constraint angles θ_k then define the maximum angular correction that can reapplied to the IPS thrust pointing vector specified at the beginning of each plan of the designed SEP profile.

$$F_k(\alpha_k, \delta_k) = \hat{p}' \cdot \hat{p} \quad (7)$$
$$\cos^{-1}(F_k(\alpha_k, \delta_k)) \leq \theta_k \quad (8)$$

The SEP thrust, profile for the 1) S-1 autonomous navigation system is then defined by a table of t_k, T_k, τ_k, α_k, $\dot{\alpha}_k$, δ_k, $\dot{\delta}_k$, α'_k, δ'_k, and θ_k for each of the planning cycles between launch and encounter, with the last three parameters used only to check that corrected SEP profiles do not violate the angular constraints imposed on the designed SEP profiles.

Linear Control Equation for SEP Profile

If the angular rates, $\dot{\alpha}_k$ and $\dot{\delta}_k$, and the thrust magnitudes T_k specified in the designed SEP profile are assumed to be fixed, then the remaining independent variables which provide control authority for the thrust vector from the spacecraft IPS are the pointing angles at the beginning of each plan, α_k and δ_k, and the thrust durations τ_κ only from the last plan in each SEP segment, since these durations are the only durations of plans within a SEP segment that are not set at the duty cycle limits. However, the last plan defined for each SEP segment, or the value of κ, is permitted to change (increase or decrease) as it becomes necessary.

It is assumed that the autonomous control system will only be used to update the SEP profile to correct for small deviations from the nominal trajectory, while any significant deviation from the nominal trajectory will require a complete redesign of the 1) S-1 trajectory and SEP profile. As such, a simple linear targeting approach seems adequate for the autonomous control system. Also, the control system will be restricted to using only those plans within a single SEP segment to correct the SEP profile at any time.

The autonomous orbit determination system computes the current best estimate of the spacecraft state at some time t, and this is integrated forward in time to provide a spacecraft state at the specified encounter time t_e using the currently available SEP profile. This present course encounter state $X_e(\alpha_k, \delta_k, \tau_\kappa)$ is a function of,

$$(\alpha_k, \delta_k, \tau_\kappa) \quad \text{for} \quad k_1 \leq k \leq \kappa, \text{ and } t < t_{k_1} < t_e$$

where the plan k_1 is the first complete plan after the time t where the best known spacecraft state has been computed. If the difference between the present course and desired encounter time spacecraft states is not below a specified tolerance threshold e, then adjustments to the parameters τ_κ, α_k, and δ_k for $k = k_1$ to $k = \kappa$, a total of $2(\kappa - k_1 + 1) + 1$ parameters, can be used to guide the spacecraft to the required target state. The desired target state $\bar{X}_e(\bar{\alpha}_k, \bar{\delta}_k, \bar{\tau}_k)$ is a function of,

$$(\bar{\alpha}_k, \bar{\delta}_k, \bar{\tau}_\kappa) \quad \text{for} \quad k = k_1 \text{ to } k = \kappa,$$

where the overbars ( ) are used to indicate the adjusted S11' profile variables that are necessary to achieve the required target state. It is these variables, $(\bar{\alpha}_k, \bar{\delta}_k, \bar{\tau}_\kappa)$ that must be determined by the autonomous control system.

For small deviations from the nominal trajectory it should not be necessary to use all of the available pointing angles to guide the spacecraft to the target state, and a subset of the pointing angles from plans $k = k_1$ to $k = \kappa$ could be used. If a strategy that attempts to correct the low thrust trajectory as soon as possible is adopted, then the DS-1 control system will be restricted to using the pointing angles from all plans from plan k_1 to plan k_2 to provide control authority to the IPS, where k_2 is restricted as follows.

$$k_1 \leq k_2 \leq \kappa \tag{9}$$

The required target state can be expanded into a Taylor series expansion about the present course encounter state and SEP profile as defined by the independent variables α_k, δ_k and τ_κ. Assuming hat a target trajectory SEP profile only has small deviations from the present course trajectory SEP profile, then retaining only the linear terms from the Taylor series expansion provides the linear control equation for the DS-1 SEP profile.

$$\Delta X_e = K \Delta s \tag{10}$$

The vector ΔX_e is the difference between the desired target state and the spacecraft state at encounter time computed from the current SEP profile.

$$\Delta X_e = \bar{X}_e(\bar{\alpha}_k, \bar{\delta}_k, \bar{\tau}_\kappa) - X_e(\alpha_k, \delta_k, \tau_\kappa) \tag{11}$$

The matrix $K(\alpha_k, \delta_k, \tau_\kappa)$ contains the first order partial derivatives of the control variables, and should be evaluated from the present course SEP profile used to compute $X_e(\alpha_k, \delta_k, \tau_\kappa)$.

$$K^T(\alpha_k, \delta_k, \tau_\kappa) = \begin{vmatrix} (\partial X_e / \partial \alpha_{k_1}) \\ (\partial X_e / \partial \delta_{k_1}) \\ (\partial X_e / \partial \alpha_{k_1+1}) \\ (\partial X_e / \partial \delta_{k_1+1}) \\ \\ (\partial X_e / \partial \alpha_{k_2}) \\ (\partial X_e / \partial \delta_{k_2}) \\ (\partial X_e / \partial \tau_\kappa) \end{vmatrix} \tag{12}$$

The operator $[\cdot]^T$ denotes the transpose of the matrix $[\cdot]$. The 'partial derivatives in the K matrix are numerically computed using finite central differences. An example is given below.

$$\frac{\partial X_e}{\partial \alpha_{k_1}} = \frac{X_e(\alpha_k, \delta_k, \tau_\kappa)|_{\alpha_{k_1}+\epsilon} - X_e(\alpha_k, \delta_k, \tau_\kappa)|_{\alpha_{k_1}-\epsilon}}{2\epsilon} \tag{13}$$

The control vector As contains the first order corrections to the control variables of the SEP profile.

$$\Delta s = \begin{vmatrix} \bar{\alpha}_{k_1} - \alpha_{k_1} \\ \bar{\delta}_{k_1} - \delta_{k_1} \\ \bar{\alpha}_{k_1+1} - \alpha_{k_1+1} \\ \bar{\delta}_{k_1+1} - \delta_{k_1+1} \\ \cdot \\ \bar{\alpha}_{k_2} - \alpha_{k_2} \\ \bar{\delta}_{k_2} - \delta_{k_2} \\ \bar{\tau}_\kappa - \tau_\kappa \end{vmatrix} \tag{14}$$

A total of $M = 2(k_2 - k_1 + 1) + 1$ variables provide control authority for the DS-1 low thrust trajectory, and As is a vector of dimension M. The two pointing angles from at least the first available plan k_i in a SEP segment, and the duration from the last plan κ of that SEP segment are always included in the search for an updated SEP profile, and $M \geq 3$ always. If N is used to denote the dimension of the target vector AX,, then K is a matrix of dimension N x M. The target vector is defined either by the three dimensional position coordinates at encounter time, or by the six dimensional state including position and velocity, so that N = 3 or N = 6 always. When targeting to the three dimensional position, the residual target vector ΔX_e is always specified in terms of target relative asymptotic coordinates in plane of the trajectory.

$$\Delta X_e^T = [\ \Delta B \cdot R \quad \Delta B\ T \ \ \Delta TOF \ ] \tag{15}$$

The target relative coordinates BR and B. T define positions in the two crosstrack directions, and TOF defines the along track position in terms of a time of flight with respect to the point of closest approach.

The corrections to the SEP profile that are needed to guide the spacecraft to the target state are solved through iterative solutions of Equation (10) for As. In the first iteration, the present course trajectory SEP profile is used to compute the matrix $K(\alpha_k, \delta_k, \tau_\kappa)$ and the encounter time state $X_e(\alpha_k, \delta_k, \tau_\kappa)$, which then provides a first order solution of the corrections As and an updated SEP profile defined by $(\bar{\alpha}_k, \bar{\delta}_k, \bar{\tau}_\kappa)$. The updated SEP profile then becomes the present course trajectory SEP profile in the next iteration, $(\alpha_k, \delta_k, \tau_\kappa) = (\bar{\alpha}_k, \bar{\delta}_k, \bar{\tau}_\kappa)$, from which the next set of SEP profile corrections are computed. If tile corrected duration of the last plan extends past its boundaries, as specified in Equation (4), the value of κ is increased or decreased as becomes necessary. This procedure is repeated until tile norm of the residual between the target state and the encounter state is within the specified threshold e.

$$|\ \Delta X_e\ | < \epsilon \tag{16}$$

A convergence criteria of 1 km in position and 10^{-5} km/s in velocity is usually sufficient.

Equation (10) is a linearized equation, and convergence of the iterations required to solve this equation are not guaranteed. However, tests have shown that when the iterative solution to the linear control equation dews not converge there is usually an insufficient number of control parameters in the control vector As. As such, when the iterative procedure does not converge within a specified finite number of iterations, more parameters are added to the control vector. More specifically, k_2 is incremented in steps of 1, and the dimension of the control vector is increased in steps of 2, by sequentially adding the two pointing angles of consecutive SEP plans in steps of one plan at a time, until a converged solution is found. An obvious failure mode of the control system then arises when $k_2 > \kappa$ and there are no more control parameters available to find a converged solution, and the ground navigation system would then be notified to redesign the SEP profile.

Solution Strategies of Control Equation

The method used to solve Equation (10) is dependent on the dimension M of the control vector As with respect to the dimension N of the residual encounter state vector ΔX_e. This results with three cases which each require different solution methods, Similar **solution** methods are also used when the angular constraints arc imposed.

Case 1. N = M

This is the simplest case where the number of equations and control parameters are identical. For each iteration, a unique solution of As from the control equation is computed from a simple inversion of the matrix K.

$$\text{As} = K^{-1}\Delta X_e \qquad (17)$$

Case 2. N > M

In the case where there are fewer control parameters than equations, the corrections As are computed from least squares solutions to Equation (10) at each iteration. That is, the corrections to the SEP profile are chosen to be the vector As that minimizes the following performance index J.

$$J = \frac{1}{2}(\Delta X_e - K\Delta s)^T(\Delta X_e - K\Delta s) \qquad (18)$$

The least squares solution to the control equation is found by minimizing J with respect to As.

$$\Delta s = (K^T K)^{-1} K^T \Delta X_e \qquad (19)$$

Note that since N = 3 or N = 6, and $M \geq 3$ always, the least squares solution is only used when targeting to a position *and* velocity at encounter time with the angles of fewer than 3 planning cycles. The converged least squares solutions only provide a minimum to the performance index and the residual encounter state ΔX_e, and the iterative search ends when this minimum is reached even though it does not necessarily lie within the threshold limit e.

Case 3. N < M

When there are more control parameters than the dimension of the target state, the solution to the control equation is chosen to be the solution that minimizes the corrections As subject to the constraint $\Delta X_e = K\Delta s$. 'l'he performance index is:

$$J(\Delta s, \lambda) = \frac{1}{2}(\Delta s^T \Delta s) + \lambda(\Delta X_e - K\Delta s) \qquad (20)$$

where the constraint has been adjoined with the Lagrange multipier λ. The first variation of $J(\Delta s, \lambda)$ with respect to As and λ is given as δJ below.

$$\begin{aligned} \delta J &= \frac{1}{2}(\delta\Delta s^T \Delta s + \Delta s^T \delta\Delta s) \\ &\quad - \lambda K \delta\Delta s + \delta\lambda(\Delta X_e - K\Delta s) \qquad (21) \end{aligned}$$

Note that $\delta\Delta s^T \Delta s = \Delta s^T \delta\Delta s$. For a minimum of $J(\Delta s, \lambda)$, the first variatiou δJ must vanish for arbitrary $\delta\Delta s$ and $\delta\lambda$, and the following two equations must be satisfied to have $\delta J = 0$.

$$\Delta s^T - \lambda K = 0 \qquad (22)$$
$$\Delta X_e - K\Delta s = 0 \qquad (23)$$

Inserting the transpose of Equation (22) into Equation (23) provides a solution for λ which can be inserted into the transpose of Equation (22) for a solution for As.

$$\lambda^T = (KK^T)^{-1}\Delta X_e \qquad (24)$$
$$\text{As} = K^T(KK^T)^{-1}\Delta X_e \qquad (25)$$

Equation (25) involves an inversion of an N x N matrix whose dimension is completely independent of the number of control parameters M in As, and therefore never exceeds a dimension of 6.

With Angular Constraints

After a converged solution for an updated SEP profile is computed from one of the above three solution methods it then becomes the new present course SEP profile. This new present course SEP profile is then checked to ensure that the thrust pointing vector at the beginning of each plan satisfies the angular constraint requirements from Equation (8). If the initial thrust pointing vector of any plan in this new SEP profile violates the angular constraint then corrections to this new SEP profile are computed, but by imposing an angular constraint equality to the pointing angles of all of the plans that violate the constraints. If the pointing angles from all of the plans from k_1 to k_2 that were included into the control vector used to compute this ucw SEP profile violate their respective angular constraints, then in addition to applying the angular constraint equality to all of these plans, k_2 is incremented by 1 to include the pointing angles of the next consecutive SEP plan to the control vector but without any angular constraint applied to this additional plan. As before, this procedure is repeated until a converged solution of an updated SEP profile where all the plans satisfy the angular constraints is found. When $k_2 > \kappa$ and no more plans are available to add to the iterative search, the ground navigation system is notified to redesign the SEP profile.

The angular constraint equality imposed on all of the plans which violate the constraint requirement in Equation (8) is as follows.

$$F_k(\bar{\alpha}_k, \bar{\delta}_k) = \hat{p}' \cdot \hat{\bar{p}} = \cos\theta_k \tag{26}$$

A first approximation of this constraint equality is made by defining an updated SEP profile which resets the pointing angles of the initial pointing vector $\hat{p}$ of all of the violating SEP plans in the present course SEP profile to a pointing vector $\hat{\bar{p}}$ that satisfies the constraint equality in Equation *(26),* that lies in the plane defined by $\hat{p}$ and the initial pointing vector of the design trajectory $\hat{p}'$, and that lies in between $\hat{p}$ and $\hat{p}'$.

$$\begin{aligned} (\hat{p}' \times \hat{p}) \cdot \hat{\bar{p}} &= 0 && (27) \\ \hat{\bar{p}} \cdot \hat{p} &= \cos\left[\theta_k - \cos^{-1}(\hat{p}' \cdot \hat{p})\right] && (28) \end{aligned}$$

This first approximation of the updated SEP profile becomes the new present course SEP profile and although it now satisfies the constraint equality, the residual encounter state vector ΔX_e is usually no longer within the specified threshold e. Further iterations are necessary to search for an updated SEP profile which both satisfies the constraint equality and provides a residual encounter state that is within the threshold limits.

The additional iterations are performed in a similar manner to the three methods already described above, except with additional equations that define the angular constraint equality, The linearized forru of the angular constraint equality for an arbitrary plan k is found by expanding Equation (26) into a Taylor series shout the new present course trajectory and retaining only the linear terms,

$$\begin{aligned} \Delta F_k &= F_k(\bar{\alpha}_k, \bar{\delta}_k) - F_k(\alpha_k, \delta_k) \\ &= A_k \Delta s \end{aligned} \tag{29}$$

The only nonzero elements elements of the vector A_k are those that correspond to the elements of As with right ascension and declination corrections for SEP plan k.

$$[A_k]_i = \begin{cases} \dfrac{\partial F_k}{\partial \alpha_k} & i = 2(k - k_1) + 1 \\[2ex] \dfrac{\partial F_k}{\partial \delta_k} & i = 2(k - k_1) + 2 \\[2ex] 0 & \text{all other } i \end{cases} \tag{30}$$

An expression like Equation (29) is necessary for all those plans that had violated the angular constraint in any of the prior converged solutions for a SEP profile. The partial derivatives are evaluated from the present course SEP profile, and are analytically represented as follows.

$$\frac{\partial F_k}{\partial \alpha_k} = \cos\delta_k' \cos\delta_k (\sin\alpha_k' \cos\alpha_k - \cos\alpha_k' \sin\alpha_k) \tag{31}$$

$$\frac{\partial F_k}{\partial \delta_k} = \sin\delta_k' \cos\delta_k - \cos\delta_k' \sin\delta_k (\cos\alpha_k' \cos\alpha_k + \sin\alpha_k' \sin\alpha_k) \tag{32}$$

It is important to note that both of these partial derivatives are equal to zero when the pointing angles are from the designed SEP profile, with $\alpha_k = \alpha_k'$ and $\delta_k = \delta_k'$, and the matrix A_k is then singular, However, the first approximation of the angular constraint which was computed from Equations (26) to (28), already satisfies the constraint defined in Equation (26), and subsequent iterations for the updated pointing vectors will not approach the design trajectory pointing vectors since the angular constraint equality would no longer be satisfied.

The linear control equation with angular constraints can then be considered to be a combination of Equations (10) and (29).

$$\Delta Y = K_A \Delta s \tag{33}$$

$$\Delta Y = \begin{bmatrix} \Delta X_e \\ \Delta F_k \\ \vdots \end{bmatrix}, \quad K_A = \begin{bmatrix} K \\ A_k \\ \vdots \end{bmatrix} \tag{34}$$

The vector ΔY and the matrix K_A include the residuals ΔY_k and the corresponding vectors A_k, respectively, for all of the plans k that have violated the angular constraint. If there were N_A plans that violated the angular constraints, then the dimension of the vector ΔY is $(N + N_A)$, and the dimension of K_A is $(N + N_A) \times M$.

In this case, the method chosen to solve Equation (33) is now dependent on the relationship of the dimension $(N + N_A)$ to the number of the parameters M, which result with three solution methods, say Cases 1A, 2A and 3A, which are analogous to Cases 1, 2, and 3 described above.

Case 1A: $(N + N_A) = M$

$$\Delta s = K_A^{-1} \Delta Y \tag{35}$$

Case 2A: $(N + N_A) > M$

$$\Delta s = (K_A^T K_A)^{-1} K_A^T \Delta Y \tag{36}$$

Case 3A: $(N + N_A) < M$

$$\Delta s = K_A^T (K_A K_A^T)^{-1} \Delta Y \tag{37}$$

As will be mentioned later, the DS-1 autonomous control system will usually be restricted to targeting only to the three dimensional coordinates in position that are required at the encounter time. As such, the minimum norm solution described in Case 3A is always used once angular constraints are included into the iterative search for the updated SEP profile.

Simulations of Targeting to a Position Only

Examples of some tests of the linear targeting strategy to a three dimensional position at encounter time for the DS-1 trajectory to McAuliffe using the 85% duty cycle SEP profile shown in Figure (2) as the designed SEP profile are shown below. The second SEP segment to McAuliffe will probably be redesigned after the IPS has been calibrated during the first SEP segment, so the tests are restricted to simulating errors and computing updated SEP profiles only for the second SEP segment before the McAuliffe encounter. The second segment of the design trajectory begins at SEP plan $k = 3$ and ends at SEP plan $k = 18$. It is assumed that the orbit determination system provides a perfect observation of the spacecraft state at any opportunity to update the SEP profile. The actual operation of the autonomous navigation system on DS-1 is simulated by considering the planning cycles as a time line of the DS-1 trajectory. The tests step through this time line starting with SEP plan $k = 3$, and assumes that the IPS has actually implemented a thrust in all prior SEP plans of the second segment that is equivalent to a duty cycle that is lower than the designed 85% duty cycle that would have guided the spacecraft to McAuliffe.

So, if the spacecraft is simulated to be at the beginning of plan k_1, the lower duty cycle is imposed on all plans of the updated SEP profile from $k = 3$ to $k = k_1 - 1$, and the autonomous control system is provided with an opportunity to update the SEP profile in as many future SEP plans with $k \geq k_1$ as is necessary. For example, when $k_1 = 3$, the SEP profile is exactly as designed and no corrections are applied. When $k_1 = 4$, an error in the duty cycle of plan $k = 3$ has been applied and plans with $k \geq 4$ are used to correct this error to maintain a trajectory that has an encounter with McAuliffe. Then, when $k_1 = 5$, in addition to the error already applied to plan 3, an identical error in the duty cycle of plan $k = 4$ of the SEP profile that was updated when $k_1 = 4$ is also applied, and SEP plans with $k_1 \geq 5$ are used to correct these errors. This process is repeated to the end of the second SEP segment.

Four specific examples are shown to illustrate how changing the minimum number of plans included in each solution affects the angular and duration corrections to the designed SEP profile, and how applying the angular constraint affects these corrections. The first three examples do not impose the angular constraint. The angular and duration corrections of the updated SEP profile with respect to the designed SEP profile from the first example are shown in Figure 3. These corrections are those computed by the autonomous control system when the search for an updated SEP profile is started with only 1 SEP plan, $k_2 = k_1$. The percentages labeled on each curve indicate the duty cycle that was actually applied by the IPS in the SEP plans with $3 \leq k < k_1$. Although the iterative search is started with the angles of the first available SEP plan, a converged solution is not always found with only one plan. For example, at least two plans $(k_2 = k_1 + 1)$ are necessary to find converged solutions when $k_1 = 4$, 5, and 6, and the applied duty cycles are less than 8370. As the applied duty cycle is reduced further more solution opportunities require at least two plans to find a converged solution. The extreme example is when the duty cycle applied to prior plans was 79%, and converged solutions required the used of three plans when $k_1 =$ 4, 5, 6, 7, and 8, and two plans when $k_1 = 9$, and 10.

Similarly, as the applied duty cycle is reduced the number of plans in the second segment gradually increases with the value of k increasing to the point

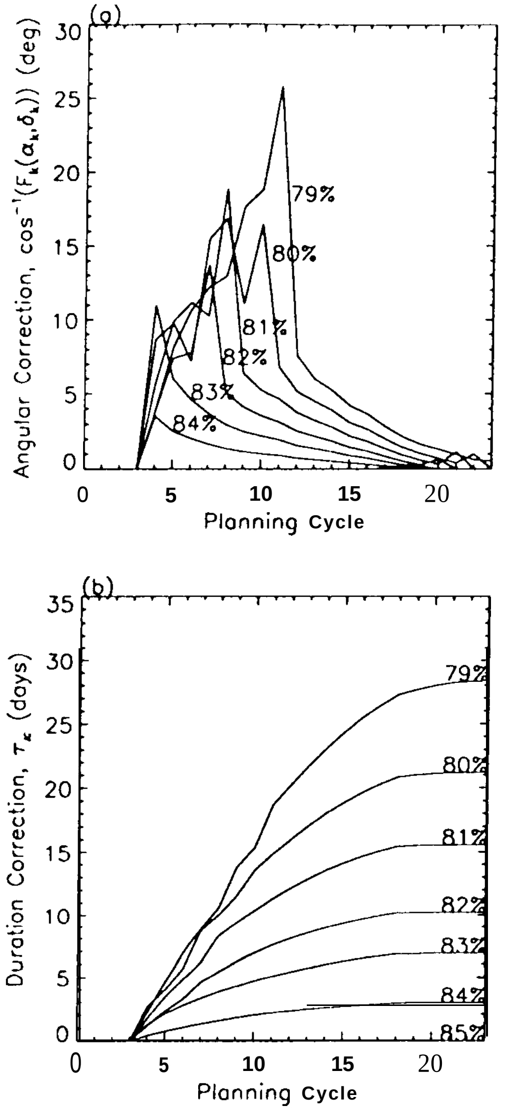

Figure 3: Angular corrections(a) and duration corrections (b) with respect to the designed SEP profile using a minimum of 1 SEP plan to correct prior errors in the SEP profile. No angular constraints are applied to the corrections.

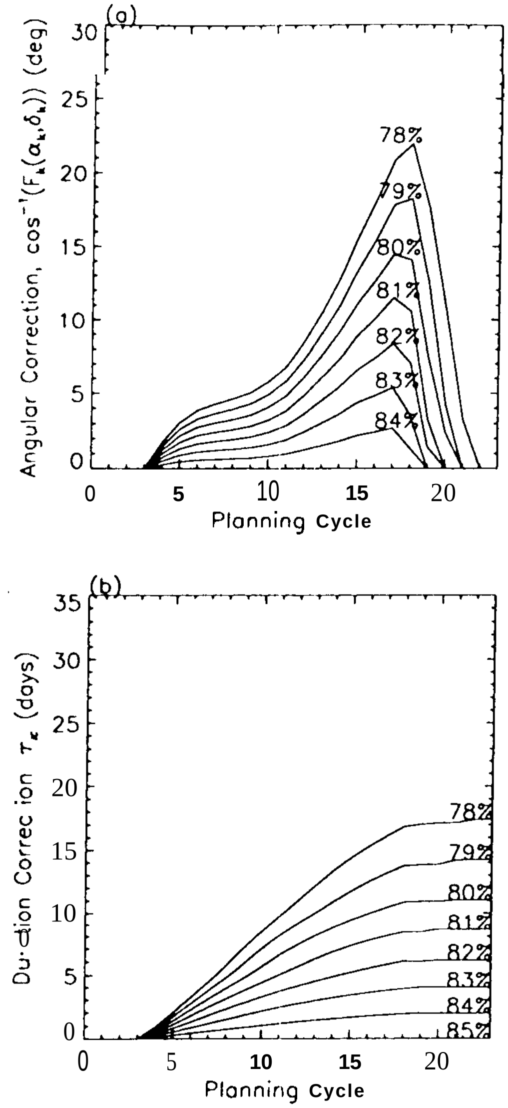

Figure 4: Same as Figure 3, but using all available SEP plans to correct prior errors in the SEP profile. No angular constraints are applied to the corrections.

where κ = 23 by the end of the simulation which applied 79% duty cycles on all prior plans. When prior plans had a duty cycle of 78% a converged solution for all of the SEP plans in the second segment could not be found because the durations eventually extended beyond plan $k = 24$ where no nominal pointing angles were specified in the designed SEP profile.

Figure 4 is similar to Figure 3 except that all available plans were used to correct any prior errors in the duty cycle and $k_2 = \kappa$ always. In this example, converged solutions were also found for all of the plans when the duty cycle applied to prior plans was 78%. This is because the duration corrections were much smaller, almost by a factor of 2, than the duration corrections when a minimum of 1 plan was used to correct errors in the duty cycle. For example, when a duty cycle of 79’%0 was applied to prior plans, the last plan of the second segment was changed from the design value of $\kappa = 18$ to κ = 23 for the example shown in Figure 3, and to κ = 20 for the example shown in Figure 4. However, reducing the duration correction also had the effect of delaying angular corrections to the plans at the end of the SEP segment, as they accumulate through each

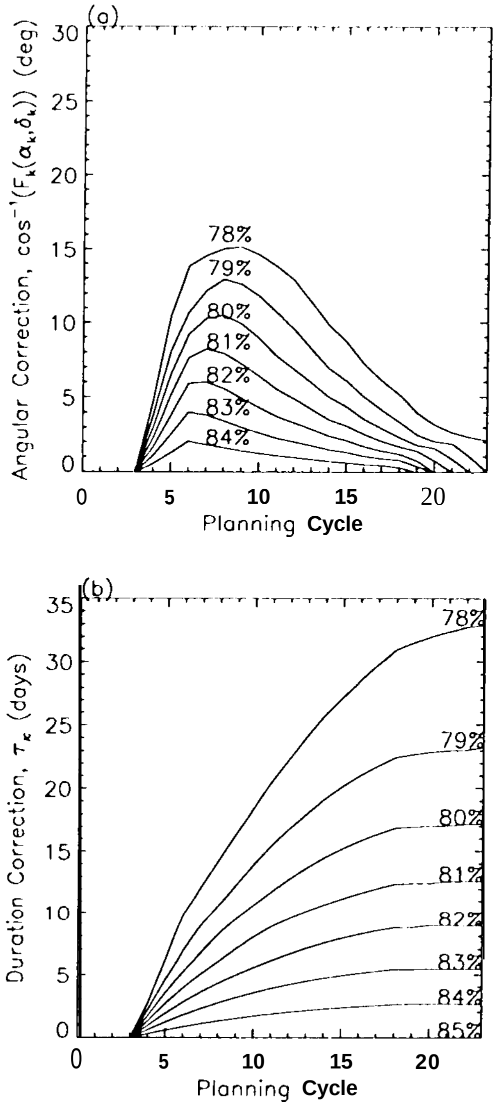

Figure 5: Same as Figure 3, but using a minimum of 3 SEP plans to correct prior errors in the SEP profile. No angular constraints are applied to the corrections,

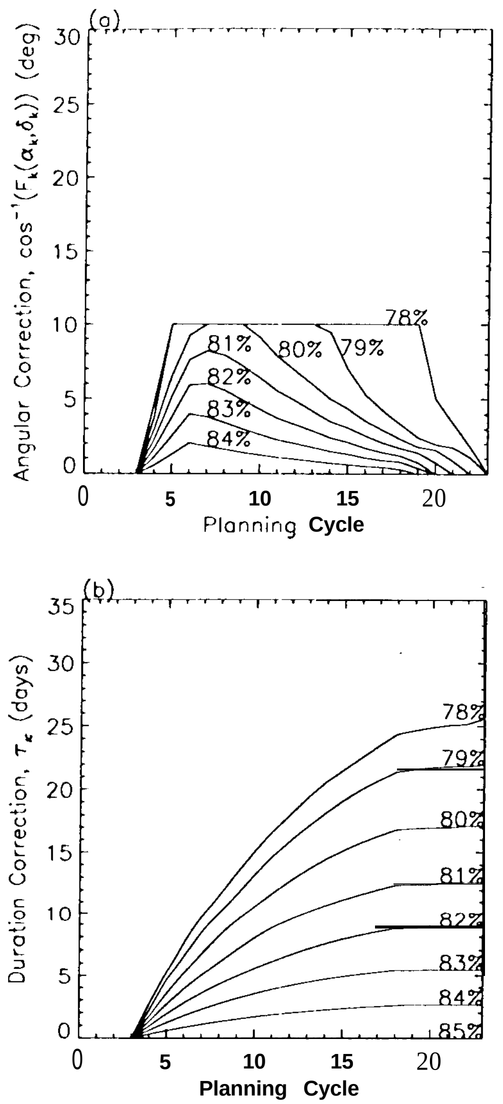

Figure 6: Same as Figure 5, but with an angular constraint of 10 degrees applied to the corrections.

update of the SEP profile from $k_1 = 3$ to $k_1 = \kappa$.

Figure 5 shows the angular corrections and duration corrections when the angles from a minimum of three segments, $k_2 = k_1 + 2$, are used to correct any errors in the duty cycle of prior SEP plans, The most significant improvement over the examples shown in Figures 3 and 4 is the reduction in the maximum angular correction of the thrust pointing vector in any plan. While the maximum angular correction in the examples shown in Figures 3 and 4 are larger than 20 degrees, in this example the maximum is only as large as approximately 15 degrees. The penalty for this improvement is larger duration corrections compared to when all the SEP plans were used to correct prior errors. However, these duration corrections are still smaller than when the angles from a minimum of 1 plan were used to update the SEP profile. In this example, when a duty cycle of 79% was applied to prior plans, the last plan of the second SEP segment is changed to $\kappa = 22$.

The designed SEP profile will usually place angular constraints on the updated SEP profiles that are of the order of 10 degrees or less. Therefore, none of the previous three examples would be suitable strategies to correct the SEP profile when applied duty cycles vary by as much as 5'%0 from the designed 85% duty cycle. Figure 6 shows a similar example to that shown in Figure 5, except that now a 10 degree angular constraint has been applied to the updated SEP profiles. The angular constraints have only been enforced when duty cycles of 81% have been applied to prior duty cycles. By applying these angular constraints there has also been a significant reduction in the durations required to correct the prior errors in the duty cycle. When a duty cycle of 78% was applied to all prior SEP plans, the last plan of the second SEP segment extended to $\kappa = 24$ when no angular corrections were imposed on the updated SEP profiles, but only extended to plan $\kappa = 22$ when the angular constraint was applied to the updated SEP profiles.

These four examples clearly dernonstratetbat using extreme strategies such as using a minimum of one plan with $k_2 = k_l$, or using all the available plans in the segment with $k_2 = \kappa$, do not provide the most desirable adjustments to the designed SEP profile. Instead, using a minimum of three plans might reconsidered as a reasonable compromise between correcting any errors as soon as possible, and reducing the angular and duration corrections to the designed SEP profile. Although the angular constraints are imposed by the physical design of the spacecraft, they also appear to improve the efficiency of the adjusted SEP profiles by reducing the duration corrections to the adjusted SEP profiles.

Targeting to only the three dimensional coordinates in position at encounter time changes the velocity and incoming asymptoteofthe spacecraft at the encounter time, and could prove to be fatal for the spacecraft trajectory to the subsequent encounters. Tests of the autonomous control system have been performed to compare the adjusted SEP profiles that would result from targeting to a six dimensional state (position and velocity, N = 6), to those that result from targeting to a three dimensional encounter state (position only, N = 3). The corrections to the thrust pointing angles and durations are much smaller when targeting to a three dimensional state and probably better suited to a linear targeting strategy. Also, for the small errors expected in the SEP thrust applied by the IPS, the changes in the velocity of the spacecraft at encounter time caused by targeting to position only, appear to be small enough to be rectified by a redesign of the SEP profile after each encounter. As such, the DS-1 autonomous control system will be restricted to linear targeting to the desired three dimensional coordinates in position at encounter time, but will maintain the capability to target to a position and velocity at encounter time. Any significant errors in the SEP thrust applied by the II'S which become energetically disadvantageous for subsequent encounters will require a redesign of the SEP profile by the ground navigation team.

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